Physics IV
ISI B.Math
Back Paper Exam : May 26 ,2015

Total Marks: 50
Time : 3 hours
Answer all questions

1. (**Marks: 4 + 4 + 2**)

(a) In the laboratory frame, an object moves with velocity (u_x, u_y) and you move with velocity v in the x-direction. What should be the value of $v(\neq 0)$ such that you also see the object move with velocity u_y in your y- direction ?

(b) The world line of a particle is described by the equations $x(t) = at + b \sin \omega t$, $y(t) = b \cos \omega t$, $z(t) = 0 \quad |b\omega| < 1$ in some inertial frame. Describe the motion and compute the components of the particle's four-velocity and four-acceleration.

(c) Show that if two four vectors have equal components in one frame, they have equal components in all frames.

2. (**Marks: 5**)

A particle has a velocity u_x in the x-direction as measured by an observer in the inertial frame S. Its velocity in the x-direction is measured to be u'_x by an observer in another inertial frame S' which is moving with a uniform velocity v in the x-direction with respect to S. The corresponding accelerations measured with respect to S and S' are given by $a_x = \frac{du_x}{dt}$ and $a'_x = \frac{du'_x}{dt'}$ respectively. Derive the relationship between a'_x and a_x.

3. (**Marks: 10**)

A particle with mass m and energy E approaches an identical particle at rest. They collide elastically in such a way that they both scatter at an angle θ relative to the incident direction. What is θ in terms of E and m? What is θ in the nonrelativistic ($E \simeq m$) and relativistic ($E \gg m$) limit ? (Note that the energies of the particles after the collision are equal , so are the magnitudes of the momenta)

4. (**Marks: 4 + 3 + 3**)

A particle of mass m is confined to a one dimensional region $0 \leq x \leq a$ with infinite potential walls. At $t = 0$, its normalized wave function is

$$\Psi(x, 0) = \sqrt{\frac{8}{5a}} \left[1 + \cos\left(\frac{\pi x}{a}\right)\right] \sin\left(\frac{\pi x}{a}\right)$$

(a) What is the wave function at a later time $t = t_0$?

(b) What is the average energy of the system at $t = 0$ and $t = t_0$?

(c) What is the probability that the particle is found in the left half of the box (i.e, in the region $0 \leq x \leq \frac{a}{2}$) at $t = t_0$?

5. (**Marks: 4 + 6**)

A one dimensional harmonic oscillator of mass m has potential energy $V(x) = \frac{1}{2}m\omega^2 x^2$.
Consider the operators $a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$
Given that $a^\dagger\psi_n = \sqrt{n+1}\psi_{n+1}$ and $a\psi_n = \sqrt{n}\psi_{n-1}$, where ψ_n is a solution of the time independent Schrödinger equation with energy E_n

(a) Given that a lowest energy ground state exists such that $a\psi_0 = 0$, find the normalized ground state wave function ψ_0 .

(b) Show that in the nth eigenstate of the harmonic oscillator, the average kinetic energy $< T >$ is equal to the average potential energy $< V >$ (the Virial theorem). What is the lowest value of the average kinetic energy that a quantum harmonic oscillator can have ? How does it contrast with the classical value ?

6. (**Marks: 5**)

A free particle of mass m moving in one dimension is known to be in the initial state

$$\psi(x, 0) = \sin(k_0 x)$$

What is $\psi(x, t)$? Is it an eigenstate of momentum ?